Given any trigonometric equation, finding a solution depends on rewriting the equation as one of the following two types.

Two fundamental types of equations.

■ **Type 1. $\sin k\,x = c$, c a constant.**

$\boldsymbol{k = 1}$. In the interval $0 \le x < 2\pi$, there are two values of x, call them x_1 and x_2, that make this equation true. In the interval $(-\infty, \infty)$, there are infinitely many values based on x_1 and infinitely many values based on x_2. The sequence based on x_1 may be written $x_1 + 2\,n\,\pi$, $n \in \mathbb{Z}$ and the sequence based on x_2 may be written $x_2 + 2\,n\,\pi$, $n \in \mathbb{Z}$.

$\boldsymbol{k \neq 1}$. An example provides the clearest explaination of the technique for solving this type of equation. Consider the equation $\boldsymbol{\sin 2\,x = \frac{\sqrt{3}}{2}}$. Since $\sin \frac{\pi}{3}$ and $\sin \frac{2\pi}{3}$ are each equal to $\frac{\sqrt{3}}{2}$, we know that the equation is satisfied when either $2\,x = \frac{\pi}{3} + 2\,n\,\pi$ or $2\,x = \frac{2\pi}{3} + 2\,n\,\pi$. Then,

(1) $2\,x = \frac{\pi}{3} + 2\,n\,\pi \Longleftrightarrow x = \frac{\pi}{6} + n\,\pi$,

and

(2) $2\,x = \frac{2\pi}{3} + 2\,n\,\pi \Longleftrightarrow x = \frac{\pi}{3} + n\,\pi$.

$\therefore\ x = \frac{\pi}{6} + n\,\pi \bigvee x = \frac{\pi}{3} + n\,\pi$.

■ **Type 2. $\sin k\,(x - \alpha) = \sin m\,(x - \beta)$.**

$\sin k\,(x-\alpha) = \sin m\,(x-\beta)$ is true when $k\,(x-\alpha) = m\,(x-\beta) + 2\,n\,\pi$. But, recall that $\sin\theta = \sin(\pi - \theta)$. So, $\sin k\,(x-\alpha) = \sin m\,(x-\beta)$ is true when $k\,(x-\alpha) = \pi - (m\,(x-\beta)) + 2\,n\,\pi$.

Example. Solve $\sin 2\left(x - \frac{\pi}{3}\right) = \sin 3\left(x - \frac{\pi}{6}\right)$.

Rewrite $\sin 2\left(x - \frac{\pi}{3}\right)$ as $\sin\left(2\,x - \frac{2\pi}{3}\right)$ and $\sin\left(3\,x - \frac{\pi}{2}\right)$. So we want to solve

$$\sin\left(2\,x - \frac{2\,\pi}{3}\right) = \sin\left(3\,x - \frac{\pi}{2}\right) \qquad (1)$$

This equationis true when

$$\left(2\,x - \frac{2\,\pi}{3}\right) = \left(3\,x - \frac{\pi}{2}\right) + 2\,n\,\pi \qquad (2)$$

or

$$\left(2x - \frac{2\pi}{3}\right) = \pi - \left(3x - \frac{\pi}{2}\right) + 2n\pi \tag{3}$$

Working with equation (1),

$$\begin{aligned}
&\left(2x - \tfrac{2\pi}{3}\right) = \left(3x - \tfrac{\pi}{2}\right) + 2n\pi \\
&\Longleftrightarrow 2x = 3x - \tfrac{\pi}{2} + 2n\pi + \tfrac{2\pi}{3} \\
&\Longleftrightarrow 2x = 3x + \tfrac{2\pi}{3} - \tfrac{\pi}{2} + 2n\pi \\
&\Longleftrightarrow 2x = 3x + \tfrac{\pi}{6} + 2n\pi \\
&\Longleftrightarrow x = -\tfrac{\pi}{6} + 2n\pi
\end{aligned}$$

Working with equation (2),

$$\begin{aligned}
&\left(2x - \tfrac{2\pi}{3}\right) = \pi - \left(3x - \tfrac{\pi}{2}\right) + 2n\pi \\
&\Longleftrightarrow 2x - \tfrac{2\pi}{3} = \pi - 3x + \tfrac{\pi}{2} + 2n\pi \\
&\Longleftrightarrow 5x = \tfrac{13\pi}{6} + 2n\pi \\
&\Longleftrightarrow x = \tfrac{13\pi}{30} + \tfrac{2n\pi}{5}
\end{aligned}$$

$$\therefore \;\; x = -\tfrac{\pi}{6} + 2n\pi \bigvee x = \tfrac{13\pi}{30} + \tfrac{2n\pi}{5}$$